PERGAMON

Microelectronics Reliability 42 (2002) 1235–1241

MICROELECTRONICS RELIABILITY

www.elsevier.com/locate/microrel

Testing process performance based on the yield: an application to the liquid-crystal display module

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Received 17 January 2002; received in revised form 18 March 2002

Abstract

Process capability indices have been introduced to provide numerical measures on whether a manufacturing process is capable of reproducing items meeting the specifications predetermined by the product designers or the consumers. Process yield is one of the most common criteria used in the manufacturing industry for measuring process performance. The formula S_{pk} has been proposed to calculate the process yield for normal processes. The formula S_{pk} provides an exact measure on the process yield. Unfortunately, the statistical properties of the estimated $\hat{S}_{pk}$ are mathematically intractable. In this paper, we apply the bootstrap simulation method to construct the lower confidence bound of S_{pk}. We then present a real-world application to the liquid-crystal display module process, to illustrate how we may apply the formula S_{pk} to actual data collected from the factories.

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1. Introduction

Process capability indices (PCIs) have been proposed to the manufacturing industry to provide numerical measures on whether a process is capable of producing product items within the specification limits predetermined by the product designers or the consumers. The PCIs were first applied to the automatic industry in Japan and America. In a purchasing contract, a minimum value of the PCI is usually specified. If the prescribed minimum value of the PCI fails to be met, the process is determined to be incapable. Otherwise, the process will be determined to be capable.

Process yield has longtime been the most common and standard criteria used in various manufacturing industries to characterize the process performance. Process yield is currently defined as the percentage of the processed product units passing the inspections. That is, the product characteristic must fall within the manufacturing tolerance. For product units rejected during the inspection (nonconforming), additional costs would be incurred to the factory for scrapping or reworking the product. All passed product units are treated equally and accepted by the producer, which requires no additional cost to the factory.

For processes involving two-sided manufacturing specifications, the process yield can be calculated as Yield $= F(\mathrm{USL}) - F(\mathrm{LSL})$, where USL and LSL are the upper and the lower specification limits, respectively, and $F(x)$ is the cumulative distribution function of the process characteristic. If the process characteristic is normally distributed, then the process yield can be expressed as Yield $= \Phi[(\mathrm{USL}-\mu)/\sigma] - \Phi[(\mathrm{LSL}-\mu)/\sigma]$, where the parameter μ is the process mean, parameter σ is the process standard deviation, and $\Phi(x)$ is the cumulative distribution function of the standard normal distribution $N(0,1)$. Based on the above expression of the process yield, Boyles [1] considered the index S_{pk} to calculate the yield for normal processes, defined as

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{\mathrm{USL}-\mu}{\sigma}\right) + \frac{1}{2}\Phi\left(\frac{\mu-\mathrm{LSL}}{\sigma}\right)\right\}.$$

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PII: S0026-2714(02)00081-1

Nomenclature

$F(x)$ the cumulative distribution function of the process characteristic
$\Phi(x)$ the cumulative distribution function of the standard normal distribution $N(0,1)$
$\{x_1, x_2, \ldots, x_n\}$ a random sample of size n taken from a process
$\{x_1^*, x_2^*, \ldots, x_n^*\}$ a bootstrap sample of size n drawn (with replacement) from the original sample $\{x_1, x_2, \ldots, x_n\}$
$\hat{\theta}^*$ an estimate of θ can be computed from the bootstrap sample
$z_{\alpha/2}$ the upper $\alpha/2$ percentile of the $N(0,1)$ distribution
$\hat{\theta}^*(i)$ the ith ordered $\hat{\theta}_i^*$, $i = 1, 2, \ldots, B$

The index S_{pk} establishes the relationship between the manufacturing specifications and the actual process performance, which provides an exact measure on the process yield. If $S_{pk} = c$, then the process yield can be expressed as Yield $= 2\Phi(3c) - 1$. The fraction of nonconforming is defined as the percentage of product items fall outside the specification limits, which can be represented as $1 - \text{Yield} = 2(1 - \Phi(3S_{pk}))$. Obviously, there is a one-to-one correspondence between S_{pk} and the process yield. Thus, S_{pk} provides an exact measure of the process yield. Note that other existing methods only provide approximate (instead of exact) yield measures. Table 1 displays the fraction of nonconforming (in ppm) as a function of the index S_{pk}, (calculated by SAS computer programs).

2. Estimating the measure S_{pk}

A natural estimator of the index S_{pk} may be obtained by replacing the unknown process mean μ by the sample mean $\overline{X} = (X_1 + X_2 + \cdots + X_n)/n$, and the unknown process standard deviation σ by the sample standard deviation, $S = [\sum(X_i - \overline{X})^2/(n-1)]^{1/2}$, respectively, which may be obtained from a stable (statistically in control) process. Thus, the natural estimator $\hat{S}_{pk}$ may be expressed as

$$\hat{S}_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{\text{USL} - \overline{X}}{S}\right) + \frac{1}{2}\Phi\left(\frac{\overline{X} - \text{LSL}}{S}\right)\right\}.$$

Unfortunately, the exact distribution of $\hat{S}_{pk}$ is mathematically intractable. Lee et al. [2] derived an approximate distribution of the estimator $\hat{S}_{pk}$ using Taylor expansion technique. They showed that the estimator $\hat{S}_{pk}$ is approximately distributed as the normal distribution. The calculation of the approximation is, however, rather messy, and cumbersome to deal with. Further, the accuracy of the approximation has not been investigated. Thus, the approximation would not be practically useful until the difficulty is overcome. For practical purpose, in this paper we use the bootstrap resampling technique to find the lower confidence bound on S_{pk}, so that practitioners/engineers can use them to perform quality testing and determine whether their process meets the preset quality requirement. We also present a case study on a liquid-crystal display module (LCM) manufacturing process to illustrate how the bootstrap lower confidence limit of S_{pk} may be applied to actual process data collected from the factory.

3. The bootstrap methodology

Efron [3,4] introduced a nonparametric, computational intensive but effective estimation method called "the bootstrap", which is a data based simulation technique for statistical inference. In particular, one can use the nonparametric bootstrap method to estimate the sampling distribution of a statistic, while assuming only that the sample is a representative of the population from which it is drawn, and that the observations are independent and identically distributed. The merit, in its simplest form, is that the nonparametric bootstrap does not rely on any distributional assumptions about the underlying population.

Suppose the set of observations $\{x_1, x_2, \ldots, x_n\}$ is a random sample of size n taken from a process. A bootstrap sample, $\{x_1^*, x_2^*, \ldots, x_n^*\}$, is a sample of size n drawn (with replacement) from the original sample $\{x_1, x_2, \ldots, x_n\}$. Hence, there are a total of n^n possible

Table 1
Various S_{pk} values and the fraction of nonconformities (in ppm)

S_{pk}	0.25	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	2.00
ppm	453 255	133 614	71 861	35 729	16 395	6934	2700	967	318	96	27	7	2	0.34	0.07	0

resamples. Thus, the bootstrap sampling is equivalent to sampling with replacement from the empirical distribution function, and the bootstrap distribution is an estimator of the distribution of $\hat{S}_{pk}$. In practice, usually only a small number of random samples out of the n^n possible resamples is drawn, the estimate is calculated for each of these, and the subsequent empirical distribution is referred to as the statistic's bootstrap distribution. Efron and Tibshirani [5] indicated that a rough minimum of 1000 bootstrap resamples is usually sufficient for computing confidence interval estimates with reasonably accuracy.

Suppose the random variable X measures process performance with respective to certain quality characteristic. The distribution of X is generally unknown. Suppose we wish to estimate some parameter, θ, that characterizes the performance of the process. An estimate of θ can be computed from the bootstrap sample, denoted as $\hat{\theta}^*$, which is called the bootstrap estimate. The resampling procedure can be performed repeatedly to obtain a certain number of bootstrap samples, for example, B times. Then, the B bootstrap estimates $\hat{\theta}_1^*, \hat{\theta}_2^*, \ldots, \hat{\theta}_B^*$, can be computed from the resamples. Research papers discussing the bootstrap methods include Efron and Gong [6], Gunter [7,8], Mooney and Duval [9], Young [10], and many others. In particular, Efron and Tibshirani [5] developed three types of bootstrap confidence interval. Those include the standard bootstrap (SB) confidence interval, the percentile bootstrap (PB) confidence interval, and the biased-corrected percentile bootstrap (BCPB) confidence interval, which are defined as follows:

(1) *Standard bootstrap:* From the B bootstrap estimates $\{\hat{\theta}_i^*\}$, $i = 1, 2, \ldots, B$, we calculate the sample average, and the sample standard deviation as

$$\bar{\theta}^* = \frac{1}{B}\sum_{i=1}^{B}\hat{\theta}_i^*,$$

$$S_{\hat{\theta}}^* = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}(\hat{\theta}_i^* - \bar{\theta}^*)^2}.$$

If the distribution of the estimate $\hat{\theta}$ is approximately normal, the $(1-\alpha)100\%$ level SB confidence interval for θ is $\hat{\theta}^* \pm z_{\alpha/2}S_{\hat{\theta}}^*$, where $z_{\alpha/2}$ is the upper $\alpha/2$ percentile of the $N(0, 1)$ distribution.

(2) *Percentile bootstrap:* From the ordered collection of $\{\hat{\theta}_i^*\}$, $i = 1, 2, \ldots, B$, the $\alpha/2$ percentage and the $(1-\alpha/2)$ percentage points are used to obtain the $(1-\alpha)100\%$ level PB confidence interval for θ, where $\hat{\theta}^*(i)$ is the ith ordered $\hat{\theta}_i^*$, $i = 1, 2, \ldots, B$.

$$[\hat{\theta}^*(\alpha/2 \times B), \hat{\theta}^*((1-\alpha/2) \times B)].$$

(3) *Biased-corrected PB:* The bootstrap distribution obtained using only a sample of the complete bootstrap distribution may be shifted higher or lower than would be expected (i.e, a biased distribution). Thus, a modified method is developed to correct for this potential bias (see Efron [4] for a complete justification of this method). First, we use the (ordered) distribution of $\hat{\theta}_i^*$ to calculate the probability $p_0 = \Pr[\hat{\theta}^*(i) \leqslant \hat{\theta}]$, $i = 1, 2, \ldots, B$, where $\hat{\theta}$ is the estimated value of θ using a random sample $\{x_1, x_2, \ldots, x_n\}$. Second, we calculate the percentile point $z_0 = \Phi^{-1}(p_0)$, and $P_L = \Phi(2z_0 - z_{\alpha/2})$, $P_U = \Phi(2z_0 + z_{\alpha/2})$, where $\Phi(.)$ is the standard normal cumulative distribution function. Finally, the BCPB confidence interval is found as $[\hat{\theta}^*(P_L B), \hat{\theta}^*(P_U B)]$.

Franklin and Wasserman [11] investigated the lower confidence bounds for the capability indices, C_p, C_{pk} and C_{pm} using the three bootstrap methods. Some simulations were conducted, and a comparison was made among the three bootstrap methods based on the parametric estimates. The simulation results indicate that for normal processes the bootstrap confidence limits perform equally well (see Chou et al. [12], Bissell [13], and Boyles [14]). And for nonnormal processes the bootstrap estimates performed significantly better than other methods. Franklin and Wasserman [11] also found that in all cases investigated the SB method performs better than the PB and the BCPB methods. Pearn and Chen [15] also applied the three bootstrap methods to find the lower confidence bounds of the yield measure S_{pk}, and compared their performance based on the coverage percentage. Their results showed that the SB performs better than the other two methods. In particular, as the sample size n exceeds 45, the coverage percentage of the 95% lower confidence bound for the SB method, is greater than 90%.

4. LCM manufacturing process

To illustrate how the bootstrap lower confidence bound of S_{pk} may be established and applied to the actual data collected from the factories, we present a case study on the liquid-crystal module manufacturing process. The case we investigated was taken from a manufacturing factory located on the Science-Based Industrial Park, Taiwan, making the LCMs. The LCM is one of the key components used in many high-tech electronic commercial devices, such as the cellular phone, the PDA (personal digital assistant), the pocket calculator, digital watch, automobile accessory visual displays, and many others. Three key components make the LCM functions properly. Those include the liquid-crystal display, the back lighting, and the peripheral (interface) system. A typical assembly drawing for the LCM product is depicted in Fig. 1.

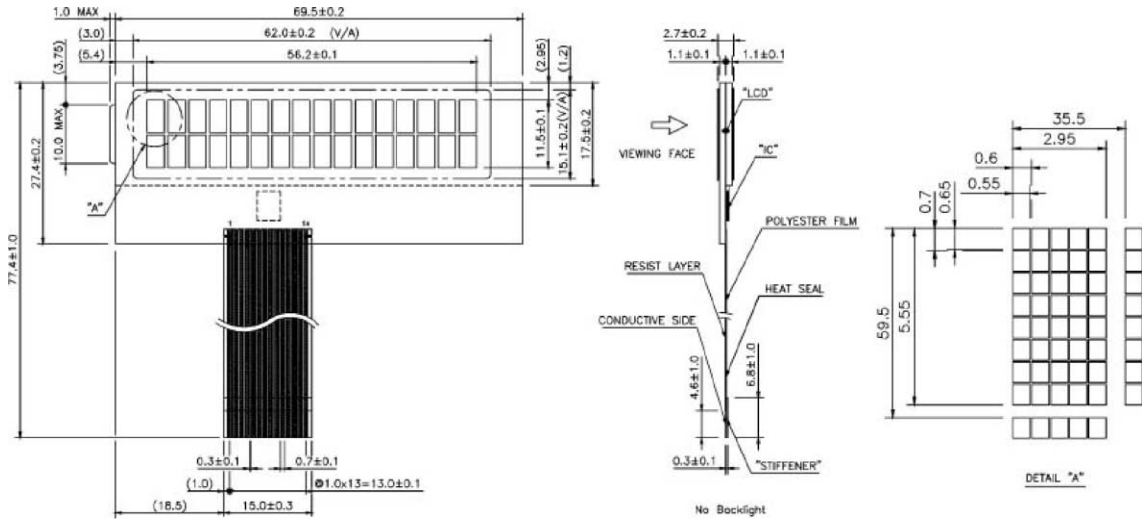

Fig. 1. The assembly drawing of the LCM product.

The mounting technology for the chip on glass (COG) makes the exposed particle overturned, with the side of circuits facing downward. Then, the electricity conduction is joined between the IC and the panel of the liquid-crystal display through the mounting material. Currently, the mounting technology of the COG is the best manufacturing technology for the LCM in terms of the mounting density. It is important to note that different mounting material requires different mounting technology of the COG.

In the factory, the manufacturing process control flow-chart of the COG is illustrated in Fig. 2. For the main bonding process, the bonding precision is an essential process parameter we focused on in our study. We investigated a particular model of the LCM product with the upper and the lower specification limits set to $USL = 15$ μm, $LSL = -15$ μm, and the target value is set to $T = 0$. If the characteristic data does not fall within the tolerance (LSL, USL), the lifetime or reliability of the LCM will be discounted. To ensure the production quality, and to satisfy the customers' requirement, the company has set the yield index $S_{pk} \geqslant 1.50$, which implies that no more than 7 ppm fraction of nonconforming for the product. If the capability requirement fails to be met, the LCM product would be seriously affected on its reliability or lifetime.

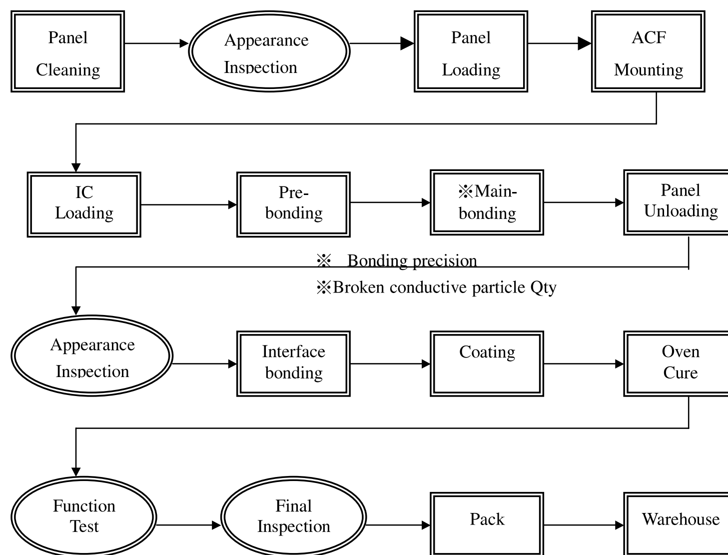

Fig. 2. Flow-chart of the COG manufacturing process.

Table 2
The collected sample data with 64 observations (unit: μm)

−0.98	4.63	0.78	−1.67
2.34	−1.97	−0.07	−2.32
2.18	3.79	0.35	−1.20
3.13	−5.58	−1.91	−1.18
−0.66	−3.16	5.01	−1.42
2.20	2.20	2.16	−2.87
−0.31	0.83	−0.86	−2.04
1.24	5.83	0.50	−0.81
3.07	−1.42	−2.42	−4.75
1.02	−6.51	1.34	1.06
0.10	5.02	−0.76	−4.84
1.28	2.19	0.16	−2.66
−0.87	−3.52	−0.05	1.11
1.47	0.28	−1.02	−9.23
−2.01	−2.30	4.26	3.41
3.34	2.85	−2.43	−1.84

4.1. The lower confidence bound

In order to evaluate the process capability based on the 95% lower confidence limit using the SB method. According to the process control plan of the COG, the random sample data in consecutive two day are collected, which are displayed in Table 2. The sample observations are obtained through the inspection, using microscope by visual, which were collected eight pieces per every 2 h. These observations were justified taken from a stable process, and the characteristic distribution is shown to be approximate normal.

Then, $B = 10\,000$ bootstrap resamples (each is of size 64) are drawn randomly from the original sample. A 95% bootstrap lower confidence limit of the SB method for S_{pk} is constructed. If the calculated bootstrap lower confidence limit is found to be smaller than the specified 1.50 index value, we would judge that the process is incapable. Quality improvement activities will be initiated. Otherwise, the process is considered to be capable. From the 10 000 bootstrap estimates $\hat{S}^*_{pk}(i)$, the sample average can be calculated as

$$\overline{S}^*_{pk} = \frac{1}{10\,000}\sum_{i=1}^{10\,000}\hat{S}^*_{pk}(i),$$

and the sample standard deviation can be obtained as

$$S^*_{\hat{S}_{pk}} = \sqrt{\frac{1}{9999}\sum_{i=1}^{10\,000}(\hat{S}^*_{pk}(i) - \overline{S}^*_{pk})^2}.$$

Thus, the 95% SB lower confidence limit for S_{pk} can be constructed as

$$\mathrm{LCB} = \bar{S}^*_{pk} - Z_{0.05} \times S^*_{\hat{S}_{pk}}.$$

The bootstrap resampling computation is rather complicated. In order to accelerate the computation, an integrated SAS computer program is developed (see Appendix A). The practitioners only need to input the manufacturing specifications, USL, LSL, a specified quality level of S_{pk} and a collected sample data of size n. The estimated value $\hat{S}_{pk}$ and the SB lower confidence bound of S_{pk} may be easily obtained. Thus, whether or not the process is capable may be determined.

For the LCM real example we investigated, Table 2 shows the simulation result with $\hat{S}_{pk} = 1.72588$ using the SB bootstrap method, the lower confidence bound on S_{pk}, was found to be 1.44244, which is less than the specified 1.50 (the minimal value of the measure to reach a positive judgment). Thus, we may not conclude that the bonding precision of the bonding process is capable. Quality improvement team must immediately initiate some improvement activities to ensure the minimal requirement met.

5. Conclusions

Bootstrap resampling method is a nonparametric, computational intensive but effective estimation method, which is a data based simulation technique for statistical inference. In particular, one can use the nonparametric bootstrap method to estimate the sampling distribution of a statistic. Bootstrap method has been widely applied to statistical process control. In this paper, we apply the bootstrap method to the process yield measure S_{pk} to obtain the confidence bounds. The proposed approach makes it feasible for the engineers to perform approximate process performance testing using the calculated S_{pk}. We also provide an efficient SAS computer program for the engineers to use. The program only requires an input of the manufacturing specification limits, and the sample data, then the estimated value $\hat{S}_{pk}$ and SB lower confidence limit of S_{pk} will be outputted quickly. In summary, bootstrap lower confidence limit of S_{pk} may be used to evaluate process capability in statistical process control.

Appendix A.

```
/*- - - - - - - - - - - - - - - - - - - -
input the specification limits, USL, LSL, and the
target value T
- - - - - - - - - - - - - - - - - - - - -*/
data para;
input usl lsl T;
cards;
15 −15 0
;
/*- - - - - - - - - - - - - - - - - - - -
```

```
store the input of the original sample data of size
n = 64
– – – – – – – – – – – – – – – – – – – –*/
data size;
set para;
array x{64} x1–x64;
array y{64} y1–y64;
retain seed 123;
do i = 1 to 64;
   input x{i} @;
end;
/* – – – – – – – – – – – – – – – – – – – -
calculate the estimated Spk based on the original
sample of n = 64
– – – – – – – – – – – – – – – – – – – –*/
omean = mean(of x1–x64);
ostd = std(of x1–x64);
c = (usl−omean)/ostd;
d = (omean−lsl)/ostd;
ESpk = 1/3*probit(0.5*probnorm(c)+0.5*prob-
norm(d));
/* – – – – – – – – – – – – – – – – – – – -
generate 10000 bootstrap samples from the original
sample of n = 64
– – – – – – – – – – – – – – – – – – – –*/
do m = 1 to 10000;
   do n = 1 to 64;
    L = int(ranuni(seed)*64+1);
    y{n} = x{L};
   end;
/* – – – – – – – – – – – – – – – – – – – -
Compute the 10000 bootstrap estimates Ŝ*pk(i)
– – – – – – – – – – – – – – – – – – – –*/
   mean = mean(of y1–y64);
   std = std(of y1–y64);
   e = (usl−mean)/std;
   f = (mean−lsl)/std;
   Spk = 1/3*probit(0.5*probnorm(e)+0.5*prob-
   norm(f));
   output;
end;
/* – – – – – – – – – – – – – – – – – – – -
Input the original sample data of n = 64 sample
observations
– – – – – – – – – – – – – – – – – – – –*/
cards;
−0.98 4.63 0.78 −1.67 2.34
−1.97 −0.07 −2.32 2.17 3.78
0.35 −1.20 3.12 −5.58 −1.91
−1.18 −0.66 −3.16 5.01 −1.42
2.20 2.20 2.16 −2.87 −0.30
0.83 −0.86 −2.04 1.24 5.83
0.50 −0.81 3.07 −1.42 −2.42
−4.75 1.02 −6.50 1.34 1.06
0.10 5.02 −0.76 −4.84 1.28
2.19 0.16 −2.66 −0.87 −3.52
−0.05 1.11 1.47 0.28 −1.02
−9.24 −2.01 −2.30 4.26 3.41
3.34 2.85 −2.43 −1.84
;
/* – – – – – – – – – – – – – – – – – – – -
Compute S̄*pk and S*Ŝpk from the 10000 bootstrap esti-
mates Ŝ*pk(i)
– – – – – – – – – – – – – – – – – – – –*/
proc univariate data = size normal plot;
   var Spk;
   output out = out1 mean = mean
   std = stddev;
/* – – – – – – – – – – – – – – – – – – – -
Calculate the lower confidence bound on the Spk
– – – – – – – – – – – – – – – – – – – –*/
data sbcl;
   set out1;
   SBLB = mean-probit(0.95)*stddev;
proc print;
   var SBLB;
run;
```

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